

Extracting CKM Phases from Angular Distributions of $B_{d,s}$ Decays into Admixtures of CP Eigenstates

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Abstract

The time-dependent angular distributions of certain $B_{d,s}$ decays into final states that are admixtures of CP-even and CP-odd configurations provide valuable information about CKM phases and hadronic parameters. We present the general formalism to accomplish this task, taking also into account penguin contributions, and illustrate it by considering a few specific decay modes. We give particular emphasis to the decay $B_d \rightarrow J/\psi \rho^0$, which can be combined with $B_s \rightarrow J/\psi \phi$ to extract the $B_d^0\text{--}\overline{B}_d^0$ mixing phase and – if penguin effects in the former mode should be sizeable – also the angle γ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0\text{--}\overline{B}_s^0$ mixing phase from $B_s \rightarrow J/\psi \phi$. Moreover, a discrete ambiguity in the extraction of the CKM angle β can be resolved, and valuable insights into $SU(3)$ -breaking effects can be obtained. Other interesting applications of the general formalism presented in this paper, involving $B_d \rightarrow \rho\rho$ and $B_{s,d} \rightarrow K^*\overline{K}^*$ decays, are also briefly noted.

1 Introduction

Studies of CP violation in the B -meson system and the determination of the three angles α , β and γ of the usual non-squashed unitarity triangle [1] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [2] are among the central targets of future B -physics experiments. During the recent years, several strategies were proposed to accomplish this task [3]. In this context, also quasi-two-body modes $B_q \rightarrow X_1 X_2$ of neutral B_q -mesons ($q \in \{d, s\}$), where both X_1 and X_2 carry spin and continue to decay through CP-conserving interactions, are of particular interest [4, 5]. In this case, the time-dependent angular distribution of the decay products of X_1 and X_2 provides valuable information. For an initially, i.e. at time $t = 0$, present B_q^0 -meson, it can be written as

$$f(\Theta, \Phi, \Psi; t) = \sum_k \mathcal{O}^{(k)}(t) g^{(k)}(\Theta, \Phi, \Psi), \quad (1)$$

where we have denoted the angles describing the kinematics of the decay products of X_1 and X_2 generically by Θ , Φ and Ψ . Note that we have to deal, in general, with an arbitrary number of such angles. The observables $\mathcal{O}^{(k)}(t)$ describing the time evolution of the angular distribution (1) can be expressed in terms of real or imaginary parts of certain bilinear combinations of decay amplitudes. In the applications discussed in this paper, we will focus on $B_q \rightarrow [X_1 X_2]_f$ decays, where X_1 and X_2 are both vector mesons, and f denotes a final-state configuration with CP eigenvalue η_f . It is convenient to analyse such modes in terms of the linear polarization amplitudes $A_0(t)$, $A_{\parallel}(t)$ and $A_{\perp}(t)$ [6]. Whereas $A_{\perp}(t)$ describes a CP-odd final-state configuration, both $A_0(t)$ and $A_{\parallel}(t)$ correspond to CP-even final-state configurations, i.e. to the CP eigenvalues -1 and $+1$, respectively. The observables of the corresponding angular distribution are given by

$$|A_f(t)|^2 \quad \text{with} \quad f \in \{0, \parallel, \perp\}, \quad (2)$$

as well as by the interference terms

$$\Re\{A_0^*(t)A_{\parallel}(t)\} \quad \text{and} \quad \Im\{A_f^*(t)A_{\perp}(t)\} \quad \text{with} \quad f \in \{0, \parallel\}. \quad (3)$$

This formalism is discussed in more detail in [7], where several explicit angular distributions can be found and appropriate weighting functions to extract their observables in an efficient way from the experimental data are given.

In the following considerations, the main role is played by neutral $B_q \rightarrow [X_1 X_2]_f$ decays, where the “unevolved” decay amplitudes can be expressed as

$$A_f = \mathcal{N}_f \left[1 - b_f e^{i\rho_f} e^{+i\omega} \right] \quad (4)$$

$$\overline{A}_f = \eta_f \mathcal{N}_f \left[1 - b_f e^{i\rho_f} e^{-i\omega} \right], \quad (5)$$

where ω denotes a CP-violating weak phase and $\mathcal{N}_f \equiv |\mathcal{N}_f| e^{i\delta_f}$. Both ρ_f and δ_f are CP-conserving strong phases. In this case, the observables (2) and (3) allow us to probe the

$B_q^0\text{--}\overline{B}_q^0$ mixing phase ϕ_q and the weak phase ω , as we will show in this paper. Concerning practical applications, ω is given by one of the angles of the unitarity triangle. However, the observables specified in (2) and (3) are not independent from one another and do not provide sufficient information to extract ϕ_q and ω , as well as the corresponding hadronic parameters, simultaneously. To this end, we have to use an additional input.

Usually, the weak phase ω is of central interest. If we fix the mixing phase ϕ_q separately, it is possible to determine ω – and interesting hadronic quantities – as a function of a *single* hadronic parameter in a *theoretically clean* way. If we determine this quantity, for instance, by comparing $B_q \rightarrow X_1 X_2$ with an $SU(3)$ -related mode, all remaining parameters, including ω , can be extracted. If we are willing to make more extensive use of flavour-symmetry arguments, it is in principle possible to determine the $B_q^0\text{--}\overline{B}_q^0$ mixing phase ϕ_q as well. An example for such a strategy is given by the decay $B_d \rightarrow J/\psi \rho^0$, which can be combined with $B_s \rightarrow J/\psi \phi$ to extract the $B_d^0\text{--}\overline{B}_d^0$ mixing phase $\phi_d = 2\beta$ and – if penguin effects in the former mode should be sizeable – also the angle γ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0\text{--}\overline{B}_s^0$ mixing phase from $B_s \rightarrow J/\psi \phi$, which is an important issue for “second-generation” B -physics experiments at hadron machines. Moreover, we may resolve a discrete ambiguity in the extraction of the CKM angle β , and may obtain valuable insights into $SU(3)$ -breaking effects. Other interesting applications of the general formalism presented in this paper, involving $B_d \rightarrow \rho\rho$ and $B_{s,d} \rightarrow K^*\overline{K}^*$ decays, are also briefly noted.

As the extraction of ω with the help of these modes involves “penguin”, i.e. flavour-changing neutral-current (FCNC) processes and relies moreover on the unitarity of the CKM matrix, it may well be affected by new physics. In such a case, discrepancies would show up with other strategies to determine this phase, for example with the theoretically clean extractions of γ making use of pure “tree” decays such as $B_s \rightarrow D_s^\pm K^\mp$. Since no FCNC processes contribute to the decay amplitudes of these modes, it is quite unlikely that they – and the extracted value of γ – are significantly affected by new physics.

The outline of this paper is as follows: in Section 2, the time-dependent observables of the $B_q \rightarrow X_1 X_2$ angular distribution are given. The strategies to extract CKM phases, as well as interesting hadronic parameters, with the help of these observables are discussed in Section 3. In Section 4, we focus on the extraction of β and γ from $B_d \rightarrow J/\psi \rho^0$ and $B_s \rightarrow J/\psi \phi$. Further applications of the formalism developed in Sections 2 and 3 are discussed in Section 5, and the conclusions are summarized in Section 6.

2 The Time Evolution of the Angular Distributions

In this section, we consider the general case of a neutral quasi-two-body decay $B_q \rightarrow [X_1 X_2]_f$ into a final-state configuration f with CP eigenvalue η_f that exhibits “unmixed” decay amplitudes of the same structure as those given in (4) and (5). If we use linear polarization states to characterize the final-state configurations as, for example, in [7], we have $f \in \{0, \parallel, \perp\}$.

At this point a comment on the angular distribution of the CP-conjugate decay $\overline{B}_q^0 \rightarrow X_1 X_2$, which is given by

$$\overline{f}(\Theta, \Phi, \Psi; t) = \sum_k \overline{\mathcal{O}}^{(k)}(t) g^{(k)}(\Theta, \Phi, \Psi), \quad (6)$$

is in order. Since the meson content of the $X_1 X_2$ states is the same whether these result from the B_q^0 or \overline{B}_q^0 decays, we may use the same generic angles Θ , Φ and Ψ to describe the angular distribution of their decay products. Within this formalism, the effects of CP transformations relating $B_q^0 \rightarrow [X_1 X_2]_f$ to $\overline{B}_q^0 \rightarrow [X_1 X_2]_f$ are taken into account by the CP eigenvalue η_f appearing in (5), and do not affect the form of $g^{(k)}(\Theta, \Phi, \Psi)$. Therefore the same functions $g^{(k)}(\Theta, \Phi, \Psi)$ are present in (1) and (6) (see also [5]).

In view of applications to B_s decays, we allow for a non-vanishing width difference $\Delta\Gamma_q \equiv \Gamma_{\text{H}}^{(q)} - \Gamma_{\text{L}}^{(q)}$ between the B_q mass eigenstates B_q^{H} (“heavy”) and B_q^{L} (“light”). In contrast to the B_d case, this width difference may be sizeable in the B_s system [8]; it may allow studies of CP violation with “untagged” B_s data samples, where one does not distinguish between initially, i.e. at time $t = 0$, present B_s^0 or \overline{B}_s^0 mesons [9]. The time evolution of the observables corresponding to (2) takes the following form:

$$|A_f(t)|^2 = \frac{1}{2} \left[R_{\text{L}}^f e^{-\Gamma_{\text{L}}^{(q)} t} + R_{\text{H}}^f e^{-\Gamma_{\text{H}}^{(q)} t} + 2 e^{-\Gamma_q t} \left\{ A_{\text{D}}^f \cos(\Delta M_q t) + A_{\text{M}}^f \sin(\Delta M_q t) \right\} \right] \quad (7)$$

$$|\overline{A}_f(t)|^2 = \frac{1}{2} \left[R_{\text{L}}^f e^{-\Gamma_{\text{L}}^{(q)} t} + R_{\text{H}}^f e^{-\Gamma_{\text{H}}^{(q)} t} - 2 e^{-\Gamma_q t} \left\{ A_{\text{D}}^f \cos(\Delta M_q t) + A_{\text{M}}^f \sin(\Delta M_q t) \right\} \right], \quad (8)$$

where $\Delta M_q \equiv M_{\text{H}}^{(q)} - M_{\text{L}}^{(q)} > 0$ denotes the mass difference between the B_q mass eigenstates, and $\Gamma_q \equiv [\Gamma_{\text{L}}^{(q)} + \Gamma_{\text{H}}^{(q)}]/2$. The quantities R_{L}^f , R_{H}^f , A_{D}^f and A_{M}^f , which are not independent from one another and satisfy the relation

$$\left(A_{\text{D}}^f\right)^2 + \left(A_{\text{M}}^f\right)^2 = R_{\text{L}}^f R_{\text{H}}^f, \quad (9)$$

are given by

$$R_{\text{L}}^f = |\mathcal{N}_f|^2 \left[(1 + \eta_f \cos \phi_q) - 2 b_f \cos \rho_f \{ \cos \omega + \eta_f \cos(\phi_q + \omega) \} + b_f^2 \{ 1 + \eta_f \cos(\phi_q + 2\omega) \} \right] \quad (10)$$

$$R_{\text{H}}^f = |\mathcal{N}_f|^2 \left[(1 - \eta_f \cos \phi_q) - 2 b_f \cos \rho_f \{ \cos \omega - \eta_f \cos(\phi_q + \omega) \} + b_f^2 \{ 1 - \eta_f \cos(\phi_q + 2\omega) \} \right] \quad (11)$$

$$A_{\text{D}}^f \equiv 2 |\mathcal{N}_f|^2 b_f \sin \rho_f \sin \omega \quad (12)$$

$$A_{\text{M}}^f = \eta_f |\mathcal{N}_f|^2 \left[\sin \phi_q - 2 b_f \cos \rho_f \sin(\phi_q + \omega) + b_f^2 \sin(\phi_q + 2\omega) \right]. \quad (13)$$

Here the phase ϕ_q denotes the CP-violating weak B_q^0 – \overline{B}_q^0 mixing phase:

$$\phi_q = \begin{cases} 2\beta & \text{for } q = d \\ -2\delta\gamma & \text{for } q = s, \end{cases} \quad (14)$$

where $2\delta\gamma \approx 0.03$ is tiny in the Standard Model because of a Cabibbo suppression of $\mathcal{O}(\lambda^2)$. This phase cancels in

$$S_f \equiv \frac{1}{2} (|A_f(0)|^2 + |\bar{A}_f(0)|^2) = \frac{1}{2} (R_L^f + R_H^f) = |\mathcal{N}_f|^2 (1 - 2b_f \cos \rho_f \cos \omega + b_f^2). \quad (15)$$

It is also interesting to note that there are no $\Delta M_q t$ terms present in the “untagged” combination

$$|A_f(t)|^2 + |\bar{A}_f(t)|^2 = R_L^f e^{-\Gamma_L^{(q)} t} + R_H^f e^{-\Gamma_H^{(q)} t}, \quad (16)$$

whereas

$$|A_f(t)|^2 - |\bar{A}_f(t)|^2 = 2 e^{-\Gamma_q t} [A_D^f \cos(\Delta M_q t) + A_M^f \sin(\Delta M_q t)]. \quad (17)$$

Because of (9), each of the $|A_f(t)|^2$ or $|\bar{A}_f(t)|^2$ ($f \in \{0, \parallel, \perp\}$) terms of the $B_q \rightarrow X_1 X_2$ angular distribution provides three independent observables, which we may choose as A_D^f , A_M^f and S_f .

The time evolution of the interference terms (3) is analogous to (16) and (17). Let us first give the expressions for the observables corresponding to (15):

$$R \equiv \frac{1}{2} [\Re\{A_0^*(0)A_{\parallel}(0)\} + \Re\{\bar{A}_0^*(0)\bar{A}_{\parallel}(0)\}] = |\mathcal{N}_0||\mathcal{N}_{\parallel}| [\cos \Delta_{0,\parallel} - \{b_0 \cos(\rho_0 - \Delta_{0,\parallel}) + b_{\parallel} \cos(\rho_{\parallel} + \Delta_{0,\parallel})\} \cos \omega + b_0 b_{\parallel} \cos(\rho_0 - \rho_{\parallel} - \Delta_{0,\parallel})] \quad (18)$$

$$I_D^f \equiv \frac{1}{2} [\Im\{A_f^*(0)A_{\perp}(0)\} + \Im\{\bar{A}_f^*(0)\bar{A}_{\perp}(0)\}] = |\mathcal{N}_f||\mathcal{N}_{\perp}| [b_f \cos(\rho_f - \Delta_{f,\perp}) - b_{\perp} \cos(\rho_{\perp} + \Delta_{f,\perp})] \sin \omega, \quad (19)$$

where the

$$\Delta_{\bar{f},f} \equiv \delta_f - \delta_{\bar{f}} \quad (20)$$

denote the differences of the CP-conserving strong phases of the amplitudes $\mathcal{N}_f \equiv e^{i\delta_f} |\mathcal{N}_f|$ and $\mathcal{N}_{\bar{f}} \equiv e^{i\delta_{\bar{f}}} |\mathcal{N}_{\bar{f}}|$. On the other hand, the rate differences corresponding to (17) take the following form:

$$\Re\{A_0^*(t)A_{\parallel}(t)\} - \Re\{\bar{A}_0^*(t)\bar{A}_{\parallel}(t)\} = 2 e^{-\Gamma_q t} [R_D \cos(\Delta M_q t) + R_M \sin(\Delta M_q t)] \quad (21)$$

$$\Im\{A_f^*(t)A_{\perp}(t)\} - \Im\{\bar{A}_f^*(t)\bar{A}_{\perp}(t)\} = 2 e^{-\Gamma_q t} [I_f \cos(\Delta M_q t) - I_M^f \sin(\Delta M_q t)], \quad (22)$$

where

$$R_D = |\mathcal{N}_0||\mathcal{N}_{\parallel}| [b_0 \sin(\rho_0 - \Delta_{0,\parallel}) + b_{\parallel} \sin(\rho_{\parallel} + \Delta_{0,\parallel})] \sin \omega \quad (23)$$

$$R_M = |\mathcal{N}_0||\mathcal{N}_{\parallel}| [\cos \Delta_{0,\parallel} \sin \phi_q - \{b_0 \cos(\rho_0 - \Delta_{0,\parallel}) + b_{\parallel} \cos(\rho_{\parallel} + \Delta_{0,\parallel})\} \sin(\phi_q + \omega) + b_0 b_{\parallel} \cos(\rho_0 - \rho_{\parallel} - \Delta_{0,\parallel}) \sin(\phi_q + 2\omega)] \quad (24)$$

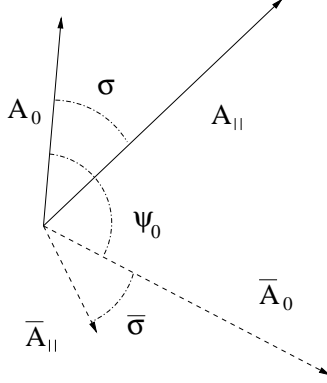


Figure 1: The amplitudes A_0 , $A_{||}$ and \bar{A}_0 , $\bar{A}_{||}$ in the complex plane.

and

$$I_f = |\mathcal{N}_f| |\mathcal{N}_\perp| \left[\sin \Delta_{f,\perp} + \{b_f \sin(\rho_f - \Delta_{f,\perp}) - b_\perp \sin(\rho_\perp + \Delta_{f,\perp})\} \cos \omega \right. \\ \left. - b_f b_\perp \sin(\rho_f - \rho_\perp - \Delta_{f,\perp}) \right] \quad (25)$$

$$I_M^f = |\mathcal{N}_f| |\mathcal{N}_\perp| \left[\cos \Delta_{f,\perp} \cos \phi_q - \{b_f \cos(\rho_f - \Delta_{f,\perp}) + b_\perp \cos(\rho_\perp + \Delta_{f,\perp})\} \cos(\phi_q + \omega) \right. \\ \left. + b_f b_\perp \cos(\rho_f - \rho_\perp - \Delta_{f,\perp}) \cos(\phi_q + 2\omega) \right]. \quad (26)$$

Note that $f \in \{0, ||\}$ in (19) and (22). The minus sign in the latter expression is due to the different CP eigenvalues of $f \in \{0, ||\}$ and $f = \perp$. If we set “0 = ||” in (23) and (24), we get expressions taking the same form as (12) and (13), which provides a nice cross check. The expressions given above generalize those derived in [7] in two respects: they take into account penguin contributions, and they allow for a sizeable value of the $B_q^0 - \bar{B}_q^0$ mixing phase ϕ_q . In the discussion of $B_s \rightarrow J/\psi \phi$ in [7], it was assumed that ϕ_s is a small phase, and terms of $\mathcal{O}(\phi_s^2)$ were neglected.

Unfortunately, not all of the observables S_f , A_D^f and A_M^f are independent from those of the interference terms (3). This can be seen by considering two different final-state configurations f and \tilde{f} . In this case, the time-dependent angular distribution provides nine observables. To be definite, let us consider the case $f = 0$ and $\tilde{f} = ||$. Then we have six observables, corresponding to S_f , A_D^f and A_M^f ($f \in \{0, ||\}$), as well as the three observables R , R_D and R_M , which are due to the real parts in (3). The measurement of S_f and A_D^f allows us to fix the magnitudes $|A_0|$, $|A_{||}|$ and $|\bar{A}_0|$, $|\bar{A}_{||}|$. Using in addition the observables R and R_D , we can determine the angle σ between the unmixed amplitudes A_0 and $A_{||}$, as well as the angle $\bar{\sigma}$ between \bar{A}_0 , $\bar{A}_{||}$ (see Fig. 1). So far, the relative orientation of the amplitudes $(A_0, A_{||})$ and $(\bar{A}_0, \bar{A}_{||})$ is not determined. However, if we use, in addition, the mixing-induced CP asymmetry A_M^0 , we are in a position to fix $\phi_q + \psi_0$, where ψ_0 denotes the angle between the amplitudes A_0 and \bar{A}_0 :

$$A_M^0 = |A_0| |\bar{A}_0| \sin(\phi_q + \psi_0). \quad (27)$$

Since the relative orientation of the amplitudes $e^{-i\phi_q}\overline{A}_\parallel$ and A_\parallel is also fixed this way, we can predict the values of the two remaining mixing-induced CP-violating observables A_M^\parallel and R_M . Consequently, only seven of the nine observables are independent from one another.

It is convenient to introduce the following “normalized” observables:

$$\hat{A}_D^f \equiv \frac{A_D^f}{S_f}, \quad \hat{A}_M^f \equiv \frac{A_M^f}{S_f}, \quad (28)$$

$$\hat{R} \equiv \frac{R}{\sqrt{S_0 S_\parallel}}, \quad \hat{R}_D \equiv \frac{R_D}{\sqrt{S_0 S_\parallel}}, \quad \hat{R}_M \equiv \frac{R_M}{\sqrt{S_0 S_\parallel}}, \quad (29)$$

$$\hat{I}_f \equiv \frac{I_f}{\sqrt{S_f S_\perp}}, \quad \hat{I}_D^f \equiv \frac{I_D^f}{\sqrt{S_f S_\perp}}, \quad \hat{I}_M^f \equiv \frac{I_M^f}{\sqrt{S_f S_\perp}}, \quad (30)$$

which have the advantage that they do not depend on the overall normalization factors $|\mathcal{N}_f|$. The observables \hat{A}_D^f and \hat{A}_M^f allow us to determine the hadronic parameters b_f and ρ_f as functions of ω and ϕ_q :

$$b_f = \sqrt{\frac{1}{k_f} \left[l_f \pm \sqrt{l_f^2 - h_f k_f} \right]} \quad (31)$$

$$2 b_f \cos \rho_f = u_f + v_f b_f^2 \quad (32)$$

$$2 b_f \sin \rho_f = \left[(1 - u_f \cos \omega) + (1 - v_f \cos \omega) b_f^2 \right] \left(\frac{\hat{A}_D^f}{\sin \omega} \right), \quad (33)$$

where

$$h_f = u_f^2 + D_f (1 - u_f \cos \omega)^2 \quad (34)$$

$$k_f = v_f^2 + D_f (1 - v_f \cos \omega)^2 \quad (35)$$

$$l_f = 2 - u_f v_f - D_f (1 - u_f \cos \omega)(1 - v_f \cos \omega), \quad (36)$$

with

$$u_f = \frac{\eta_f \hat{A}_M^f - \sin \phi_q}{\eta_f \hat{A}_M^f \cos \omega - \sin(\phi_q + \omega)} \quad (37)$$

$$v_f = \frac{\eta_f \hat{A}_M^f - \sin(\phi_q + 2\omega)}{\eta_f \hat{A}_M^f \cos \omega - \sin(\phi_q + \omega)} \quad (38)$$

and

$$D_f = \left(\frac{\hat{A}_D^f}{\sin \omega} \right)^2. \quad (39)$$

It should be emphasized that no approximations were made in order to derive these expressions. If we consider, in addition to \hat{A}_D^f and \hat{A}_M^f , either the observables specified in (29) or those given in (30), we obtain seven normalized observables, which depend on five hadronic parameters (b_f , ρ_f , $b_{\bar{f}}$, $\rho_{\bar{f}}$ and $\Delta_{\bar{f},f}$), as well as on the two CP-violating weak phases ϕ_q and ω . However, only five of the seven observables are independent from one another, so that we have not sufficient observables at our disposal to extract these parameters simultaneously. To accomplish this task, we have to make use, for example, of another decay that can be related to $B_q \rightarrow X_1 X_2$ through flavour-symmetry arguments. On the other hand, if we use ϕ_q and ω as an input, all hadronic parameters describing the decay $B_q \rightarrow X_1 X_2$ can be extracted *without* any additional assumption, thereby providing valuable insights into hadronic physics and a very fertile testing ground for model calculations of $B_q \rightarrow X_1 X_2$. The measurement of the angular distributions discussed in this paper requires high statistics and can probably only be performed at “second-generation” B -physics experiments at hadron machines, such as LHCb or BTeV, where also decays of B_s -mesons can be studied. Since several promising strategies to extract the weak phases ϕ_q and ω at such experiments were already proposed (see, for example, [3]), it may indeed be an interesting alternative to use measurements of angular distributions not to extract CKM phases, but to explore hadronic physics.

In practical applications, the parameters b_f typically measure the ratio of “penguin” to “tree” contributions. Applying the Bander–Silverman–Soni mechanism [10], and following the formalism developed in [11, 12], which makes use – among other things – of the “factorization hypothesis”, we obtain for various classes of B decays

$$b_f \equiv b, \quad \rho_f \equiv \rho \quad \forall f \in \{0, \parallel, \perp\}. \quad (40)$$

The main reason for these relations is that the form factors, which depend on the final-state configuration f , cancel in the ratios b_f of “penguin” to “tree” contributions. Although non-factorizable contributions are expected to play an important role, thereby affecting (40), it is interesting to investigate the implications of these relations on the observables of the angular distributions in more detail. If we introduce

$$\hat{A}_D \equiv \frac{2b \sin \rho \sin \omega}{1 - 2b \cos \rho \cos \omega + b^2} \quad (41)$$

$$\hat{A}_M = \frac{\sin \phi_q - 2b \cos \rho \sin(\phi_q + \omega) + b^2 \sin(\phi_q + 2\omega)}{1 - 2b \cos \rho \cos \omega + b^2}, \quad (42)$$

we obtain

$$\hat{A}_D^f = \hat{A}_D, \quad \hat{A}_M^f = \eta_f \hat{A}_M, \quad (43)$$

$$\hat{R} = \cos \Delta_{0,\parallel}, \quad \hat{R}_D = \hat{A}_D \cos \Delta_{0,\parallel}, \quad \hat{R}_M = \hat{A}_M \cos \Delta_{0,\parallel}, \quad (44)$$

$$\hat{I}_f = \sin \Delta_{f,\perp} \quad \hat{I}_D^f = \hat{A}_D \sin \Delta_{f,\perp}, \quad (45)$$

$$I \equiv \frac{\hat{I}_M^f}{\cos \Delta_{f,\perp}} = \left[\frac{\cos \phi_q - 2b \cos \rho \cos(\phi_q + \omega) + b^2 \cos(\phi_q + 2\omega)}{1 - 2b \cos \rho \cos \omega + b^2} \right], \quad (46)$$

where

$$\left(\hat{A}_D\right)^2 + \left(\hat{A}_M\right)^2 + I^2 = 1. \quad (47)$$

These relations provide an interesting test of whether (40) is realized in the decay $B_q \rightarrow X_1 X_2$. Note that \hat{I}_M^f does not – in contrast to (45) – vanish for trivial values of $\Delta_{f,\perp}$.

3 Extracting CKM Phases and Hadronic Parameters

Let us now focus on the extraction of CKM phases from the observables of the $B_q \rightarrow X_1 X_2$ angular distribution. As we have already noted, to this end, we have to employ an additional input, since we have only five independent normalized observables at our disposal, which depend on seven “unknowns”. Although it would be desirable to determine ϕ_q and ω simultaneously, usually only the CKM phase ω is of central interest.

The $B_s^0\text{--}\overline{B}_s^0$ mixing phase $\phi_s \equiv -2\delta\gamma = 2 \arg(V_{ts}^* V_{tb})$ is negligibly small in the Standard Model. It can be probed – and in principle even determined – with the help of the decay $B_s \rightarrow J/\psi \phi$ (see, for example, [7]). Large CP-violating effects in this decay would signal that $2\delta\gamma$ is not tiny, and would be a strong indication for new-physics contributions to $B_s^0\text{--}\overline{B}_s^0$ mixing. On the other hand, the $B_d^0\text{--}\overline{B}_d^0$ mixing phase $\phi_d = 2\beta$ can be fixed in a reliable way through the “gold-plated” mode $B_d \rightarrow J/\psi K_S$ [13]. Strictly speaking, mixing-induced CP violation in $B_d \rightarrow J/\psi K_S$ probes $\sin(2\beta + \phi_K)$, where ϕ_K is related to the weak $K^0\text{--}\overline{K}^0$ mixing phase and is negligibly small in the Standard Model. Because of the small value of the CP-violating parameter ε_K of the neutral kaon system, ϕ_K can only be affected by very contrived models of new physics [14]. A measurement of mixing-induced CP violation in $B_d \rightarrow J/\psi K_S$ allows us to fix $\phi_d = 2\beta$ only up to a twofold ambiguity. Several strategies to resolve this ambiguity were proposed in the literature [15], which should be feasible for “second-generation” B -physics experiments. As we will see in the following section, also the decay $B_d \rightarrow J/\psi \rho^0$, in combination with $B_s \rightarrow J/\psi \phi$, allows us to accomplish this task.

If we use ϕ_q thus determined as an input and consider, in addition to \hat{A}_D^f and \hat{A}_M^f , either the observables specified in (29) or those given in (30), we can determine ω as a function of a *single* hadronic parameter. Let us, for the moment, focus on the latter case, i.e. on the observables \hat{A}_D^f , \hat{A}_M^f , \hat{A}_D^\perp , \hat{A}_M^\perp and \hat{I}_f , \hat{I}_D^f , \hat{I}_M^f for a given final-state configuration $f \in \{0, \parallel\}$. Since $|\mathcal{N}_f|$ and $|\mathcal{N}_\perp|$ cancel in these quantities, they depend only on the hadronic parameters b_f , ρ_f , b_\perp , ρ_\perp , $\Delta_{f,\perp}$, as well as on the weak phases ω and ϕ_q . Consequently, we have seven observables at our disposal, which depend on seven “unknowns”. However, only five of the seven observables are independent from one another, as we have discussed in the previous section. If we use ϕ_q as an input, we can, for instance, obtain ω and b_f , ρ_f , b_\perp , ρ_\perp as functions of the strong phase difference $\Delta_{f,\perp}$ in a *theoretically clean* way. Although the following discussion deals with $\Delta_{f,\perp}$, we can also replace this quantity by another hadronic parameter of our choice. If we fix $\Delta_{f,\perp}$, for example, by comparing $B_q \rightarrow X_1 X_2$ with an $SU(3)$ -related mode, all parameters can be extracted. Using in addition the observables S_f , we can also determine the normalization factors $|N_f|$. Comparing them with those of the $SU(3)$ -related mode used to fix $\Delta_{f,\perp}$,

we can obtain valuable insights into $SU(3)$ -breaking corrections. The observables that we have not used so far can be used to resolve discrete ambiguities, arising typically in the extraction of these parameters.

Let us now give the formulae to implement this approach in a mathematical way. The general expression for the observable \hat{I}_f (see (25) and (30)) leads to the equation

$$A_f \sin \Delta_{f,\perp} + B_f \cos \Delta_{f,\perp} = C_f, \quad (48)$$

where

$$A_f = \frac{1}{N_f} \left[1 - (b_f \cos \rho_f + b_\perp \cos \rho_\perp) \cos \omega + b_f b_\perp (\cos \rho_f \cos \rho_\perp + \sin \rho_f \sin \rho_\perp) \right] \quad (49)$$

$$B_f = \frac{1}{N_f} \left[(b_f \sin \rho_f - b_\perp \sin \rho_\perp) \cos \omega - b_f b_\perp (\sin \rho_f \cos \rho_\perp - \cos \rho_f \sin \rho_\perp) \right] \quad (50)$$

$$C_f = \hat{I}_f, \quad (51)$$

with

$$N_f = \sqrt{(1 - 2 b_f \cos \rho_f \cos \omega + b_f^2)(1 - 2 b_\perp \cos \rho_\perp \cos \omega + b_\perp^2)}. \quad (52)$$

The solution of (48) is straightforward, and is given as follows:

$$\sin \Delta_{f,\perp} = \frac{A_f C_f \pm \sqrt{(A_f^2 + B_f^2 - C_f^2) B_f^2}}{A_f^2 + B_f^2}, \quad \cos \Delta_{f,\perp} = \frac{C_f - A_f \sin \Delta_{f,\perp}}{B_f}. \quad (53)$$

If we insert b_f and ρ_f , determined as functions of ω and ϕ_q with the help of (31)–(33), into the expressions given above, we can – for a given value of ϕ_q – determine $\Delta_{f,\perp}$ as a function of ω . It should be emphasized that the relation between $\Delta_{f,\perp}$, ω and ϕ_q obtained this way is valid *exactly*. Using \hat{I}_D^f or \hat{I}_M^f instead of \hat{I}_f would lead to the same relation, since these observables are not independent from \hat{I}_f .

Alternatively, we may use the observables (29) instead of (30). The general expression for \hat{R} (see (18) and (29)) implies an equation similar to (48), where A_f , B_f and C_f have to be replaced through

$$A = \frac{1}{N} \left[(b_\parallel \sin \rho_\parallel - b_0 \sin \rho_0) \cos \omega + b_0 b_\parallel (\sin \rho_0 \cos \rho_\parallel - \cos \rho_0 \sin \rho_\parallel) \right] \quad (54)$$

$$B = \frac{1}{N} \left[1 - (b_0 \cos \rho_0 + b_\parallel \cos \rho_\parallel) \cos \omega + b_0 b_\parallel (\cos \rho_0 \cos \rho_\parallel + \sin \rho_0 \sin \rho_\parallel) \right] \quad (55)$$

$$C = \hat{R}, \quad (56)$$

where

$$N = \sqrt{(1 - 2 b_0 \cos \rho_0 \cos \omega + b_0^2)(1 - 2 b_\parallel \cos \rho_\parallel \cos \omega + b_\parallel^2)}. \quad (57)$$

Obviously, the most efficient strategy of combining the observables provided by the $B_q \rightarrow X_1 X_2$ angular distribution depends on their actually measured values.

If we are willing to make more extensive use of flavour-symmetry arguments than just to fix the strong phase difference $\Delta_{\bar{f},f}$, it is in principle possible to determine also the $B_q^0\text{--}\overline{B}_q^0$ mixing phase ϕ_q . In the following section, we will have a closer look at the decay $B_d \rightarrow J/\psi \rho^0$, which can be related to $B_s \rightarrow J/\psi \phi$ through $SU(3)$ arguments and a certain dynamical assumption concerning “exchange” and “penguin annihilation” topologies. However, before we turn to these modes, which allow the simultaneous extraction of $\phi_d = 2\beta$ and γ , let us first give two useful expressions for the observables \hat{R} and \hat{I}_f . Since the parameters b_f measure typically the importance of “penguin” topologies in comparison with current–current contributions, they may not be too large. If we eliminate the hadronic parameters b_f and ρ_f in \hat{R} and \hat{I}_f with the help of the observables \hat{A}_D^f and \hat{A}_M^f and keep only the leading-order terms in b_f , we obtain

$$\hat{R} \approx \cos \Delta_{0,\parallel} - \frac{1}{2} (\hat{A}_D^0 - \hat{A}_D^\parallel) \frac{\sin \Delta_{0,\parallel}}{\tan \omega} \quad (58)$$

$$\hat{I}_f \approx \sin \Delta_{f,\perp} + \frac{1}{2} (\hat{A}_D^f - \hat{A}_D^\perp) \frac{\cos \Delta_{f,\perp}}{\tan \omega}, \quad (59)$$

allowing us to determine ω if the strong phase differences $\Delta_{0,\parallel}$ or $\Delta_{f,\perp}$ are known. Interestingly, the leading-order expressions (58) and (59) do not depend on the $B_q^0\text{--}\overline{B}_q^0$ mixing phase ϕ_q . A possible disadvantage of \hat{R} is that ω enters in combination with $\sin \Delta_{0,\parallel}$. Since $\Delta_{0,\parallel}$ is a difference of CP-conserving strong phases (see (20)), it may be small, thereby weakening the sensitivity of these observables on ω . The situation concerning this point is very different in the case of the observables \hat{I}_f , which allow us to determine ω even in the case of $\Delta_{f,\perp} \in \{0^\circ, 180^\circ\}$.

4 Extracting β and γ from $B_d \rightarrow J/\psi \rho^0$ and $B_s \rightarrow J/\psi \phi$

If we combine the observables describing the time-dependent angular distribution of the decay $B_d \rightarrow J/\psi[\rightarrow l^+l^-]\rho^0[\rightarrow \pi^+\pi^-]$ with those of $B_s \rightarrow J/\psi[\rightarrow l^+l^-]\phi[\rightarrow K^+K^-]$, we may extract the $B_d^0\text{--}\overline{B}_d^0$ mixing phase $\phi_d = 2\beta$ and the angle γ of the unitarity triangle. The $B_d \rightarrow J/\psi \rho^0$ angular distribution can be obtained straightforwardly from the $B_s \rightarrow J/\psi \phi$ case, which has been discussed in detail in [7], by performing appropriate replacements of kinematical variables.

The decay $B_d^0 \rightarrow J/\psi \rho^0$ originates from $\bar{b} \rightarrow \bar{c}c\bar{d}$ quark-level transitions; the structure of its decay amplitude is completely analogous to the one of $B_s^0 \rightarrow J/\psi K_S^0$ (see [16]). For a given final-state configuration f with CP eigenvalue η_f , we have

$$A(B_d^0 \rightarrow [J/\psi \rho^0]_f) = \lambda_c^{(d)} [A_{cc}^{(c)f} + A_{\text{pen}}^{(c)f}] + \lambda_u^{(d)} A_{\text{pen}}^{(u)f} + \lambda_t^{(d)} A_{\text{pen}}^{(t)f}, \quad (60)$$

where $A_{cc}^{(c)f}$ is due to current–current contributions, and the amplitudes $A_{\text{pen}}^{(q)f}$ describe penguin topologies with internal q quarks ($q \in \{u, c, t\}$). These penguin amplitudes take

into account both QCD and electroweak penguin contributions, and

$$\lambda_q^{(d)} \equiv V_{qd} V_{qb}^* \quad (61)$$

are the usual CKM factors. Employing the unitarity of the CKM matrix and the Wolfenstein parametrization [17], generalized to include non-leading terms in λ [18], we obtain

$$A(B_d^0 \rightarrow [J/\psi \rho^0]_f) = -\lambda \mathcal{A}_f \left[1 - a_f e^{i\theta_f} e^{i\gamma} \right], \quad (62)$$

where

$$\mathcal{A}_f \equiv \lambda^2 A \left[A_{\text{cc}}^{(c)f} + A_{\text{pen}}^{(ct)f} \right], \quad (63)$$

with $A_{\text{pen}}^{(ct)f} \equiv A_{\text{pen}}^{(c)f} - A_{\text{pen}}^{(t)f}$, and

$$a_f e^{i\theta_f} \equiv R_b \left(1 - \frac{\lambda^2}{2} \right) \left[\frac{A_{\text{pen}}^{(ut)f}}{A_{\text{cc}}^{(c)f} + A_{\text{pen}}^{(ct)f}} \right]. \quad (64)$$

The quantity $A_{\text{pen}}^{(ut)f}$ is defined in analogy to $A_{\text{pen}}^{(ct)f}$, and the relevant CKM factors are given as follows:

$$\lambda \equiv |V_{us}| = 0.22, \quad A \equiv \frac{1}{\lambda^2} |V_{cb}| = 0.81 \pm 0.06, \quad R_b \equiv \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.41 \pm 0.07. \quad (65)$$

It should be emphasized that (62) is a completely general parametrization of the $B_d^0 \rightarrow J/\psi \rho^0$ decay amplitude within the Standard Model, relying only on the unitarity of the CKM matrix. In particular, this expression takes also into account final-state-interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges [19]. Comparing (62) with (4), we observe that

$$\mathcal{N}_f = -\lambda \mathcal{A}_f, \quad b_f = a_f, \quad \rho_f = \theta_f, \quad \omega = \gamma. \quad (66)$$

Let us now turn to $B_s^0 \rightarrow J/\psi \phi$. Using the same notation as in (62), we have

$$A(B_s^0 \rightarrow [J/\psi \phi]_f) = \left(1 - \frac{\lambda^2}{2} \right) \mathcal{A}'_f \left[1 + \epsilon a'_f e^{i\theta'_f} e^{i\gamma} \right], \quad (67)$$

where \mathcal{A}'_f and $a'_f e^{i\theta'_f}$ take the same form as (63) and (64), respectively, and

$$\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2}. \quad (68)$$

The primes remind us that we are dealing with a $\bar{b} \rightarrow \bar{s}$ transition. Consequently, if we compare (67) with (4), we obtain

$$\mathcal{N}_f = \left(1 - \frac{\lambda^2}{2} \right) \mathcal{A}'_f, \quad b_f = \epsilon a'_f, \quad \rho_f = \theta'_f + 180^\circ, \quad \omega = \gamma. \quad (69)$$

The $B_s \rightarrow J/\psi \phi$ and $B_d \rightarrow J/\psi \rho^0$ observables can be related to each other through

$$|\mathcal{A}'_f| = \sqrt{2} |\mathcal{A}_f| \quad (70)$$

$$\Delta'_{\bar{f},f} = \Delta_{\bar{f},f} \quad (71)$$

$$a'_f = a_f, \quad \theta'_f = \theta_f, \quad (72)$$

where the factor of $\sqrt{2}$ is due to the ρ^0 wave function. These relations rely both on the $SU(3)$ flavour symmetry of strong interactions and on the neglect of certain “exchange” and “penguin annihilation” topologies. Although such topologies, which can be probed, for example, through $B_s \rightarrow \rho^+ \rho^-$, $D^{*+} D^{*-}$ decays, are usually expected to play a very minor role, they may in principle be enhanced through final-state-interaction effects [20]. For the following considerations, it is useful to introduce the quantities

$$H_f \equiv \frac{1}{\epsilon} \left(\frac{|\mathcal{A}'_f|}{|\mathcal{A}_f|} \right)^2 \frac{S_f}{S'_f} = \frac{1 - 2 a_f \cos \theta_f \cos \gamma + a_f^2}{1 + 2 \epsilon a'_f \cos \theta'_f \cos \gamma + \epsilon^2 a_f'^2}, \quad (73)$$

which can be fixed through the “untagged” $B_d \rightarrow J/\psi \rho^0$ and $B_s \rightarrow J/\psi \phi$ observables with the help of (70). Consequently, each of the linear polarization states $f \in \{0, \parallel, \perp\}$ provides the following three observables:

$$H_f, \hat{A}_D^f, \hat{A}_M^f. \quad (74)$$

Applying (72) to (73), these observables depend only on the hadronic parameters a_f and θ_f , as well as on the $B_d^0 - \overline{B}_d^0$ mixing phase $\phi_d = 2\beta$ and the angle γ of the unitarity triangle. If we choose two different linear polarization states, the observables (74) allow us to determine the corresponding hadronic parameters and β and γ simultaneously.

This approach can be implemented in a mathematical way as follows: if we consider a given final-state configuration f and combine the observables H_f and \hat{A}_D^f , which do not depend on ϕ_d , with each other, we can determine a_f and θ_f as functions of γ :

$$a_f = \sqrt{p_f \pm \sqrt{p_f^2 - q_f}} \quad (75)$$

$$2 a_f \cos \theta_f = \frac{1 - H_f + (1 - \epsilon^2 H_f) a_f^2}{(1 + \epsilon H_f) \cos \gamma} \quad (76)$$

$$2 a_f \sin \theta_f = \left[\frac{(1 + \epsilon)(1 + \epsilon a_f^2) H_f}{(1 + \epsilon H_f)} \right] \left(\frac{\hat{A}_D^f}{\sin \gamma} \right), \quad (77)$$

where

$$p_f = \frac{[2(1 + \epsilon H_f)^2 \cos^2 \gamma - (1 - H_f)(1 - \epsilon^2 H_f)] \sin^2 \gamma - \epsilon_f E_f}{(1 - \epsilon^2 H_f)^2 \sin^2 \gamma + \epsilon^2 E_f} \quad (78)$$

$$q_f = \frac{(1 - H_f)^2 \sin^2 \gamma + E_f}{(1 - \epsilon^2 H_f)^2 \sin^2 \gamma + \epsilon^2 E_f}, \quad (79)$$

with

$$E_f = \left[(1 + \epsilon) H_f \hat{A}_D^f \cos \gamma \right]^2. \quad (80)$$

These expressions allow us to eliminate the hadronic parameters a_f and θ_f in the mixing-induced CP asymmetry \hat{A}_M^f , thereby fixing a contour in the γ - ϕ_d plane, which is related to

$$\tilde{A}_f \sin \phi_d + \tilde{B}_f \cos \phi_d = \tilde{C}_f, \quad (81)$$

with

$$\tilde{A}_f = 1 - 2 a_f \cos \theta_f \cos \gamma + a_f^2 \cos 2\gamma \quad (82)$$

$$\tilde{B}_f = -2 a_f \cos \theta_f \sin \gamma + a_f^2 \sin 2\gamma \quad (83)$$

$$\tilde{C}_f = \left(1 - 2 a_f \cos \theta_f \cos \gamma + a_f^2 \right) (\eta_f \hat{A}_M^f). \quad (84)$$

The solution of (81) has already been given in (53). If we consider two different final-state configurations f and \tilde{f} , we obtain two different contours in the γ - ϕ_d plane; their intersection allows us to determine both γ and $\phi_d = 2\beta$. Using, in addition, the observables (29) or (30) – depending on which final-state configurations f and \tilde{f} we consider – we may resolve discrete ambiguities, arising typically in the extraction of ϕ_d and γ .

Because of the strong suppression of $a'_{f'}$ through $\epsilon = 0.05$ in (73), this approach is essentially unaffected by possible corrections to (72), and relies predominantly on the relation (70). If we insert the values of ϕ_d and γ thus determined into the expressions for the observables of the third linear polarization state f' , which has not been used so far, its hadronic parameters $|\mathcal{A}_{f'}|$, $a_{f'}$ and $\theta_{f'}$ can also be determined. Comparing $|\mathcal{A}_{f'}|$ with the $B_s \rightarrow J/\psi \phi$ parameter $|\mathcal{A}'_{f'}|$, we can obtain valuable insights into the validity of (70). Moreover, several other interesting cross checks can be performed with the many observables of the angular distributions. Because of our poor understanding of the hadronization dynamics of non-leptonic B decays, only the “factorization” approximation can be used for the time being to estimate factorizable $SU(3)$ -breaking corrections to (70). Explicit expressions for the $B_s \rightarrow J/\psi \phi$ observables can be found in [7], and $SU(3)$ -breaking effects in the corresponding form factors were studied in [21]. However, also non-factorizable effects are expected to play an important role, and experimental insights into these issues would be very helpful to find a better theoretical description.

The simultaneous extraction of ϕ_d and γ discussed above works only if the hadronic parameters a_f and θ_f are sufficiently different from each other for two different final-state configurations f . If, for example, (40) should apply to $B_d \rightarrow J/\psi \rho^0$ – which seems to be quite unlikely – the B_d^0 - \overline{B}_d^0 mixing phase has to be fixed separately in order to determine γ . In this case, each linear polarization state $f \in \{0, \parallel, \perp\}$ provides a strategy to extract γ that is completely analogous to the one proposed in [16], which makes use of $B_{s(d)} \rightarrow J/\psi K_S$ decays. If we combine H_f with \hat{A}_M^f , we obtain

$$a_f = \sqrt{\frac{H_f - 1 + u_f (1 + \epsilon H_f) \cos \gamma}{1 - v_f (1 + \epsilon H_f) \cos \gamma - \epsilon^2 H_f}}. \quad (85)$$

The intersection of the contours in the γ - a_f plane described by this expression with those related to (31) allows us to determine γ and a_f .

If we use ϕ_d as an input in order to extract γ from $B_d \rightarrow J/\psi \rho^0$, it is, however, more favourable to follow the approach discussed in the previous section, i.e. to use (53), and to fix $\Delta_{f,\perp}$ (or $\Delta_{0,\parallel}$) through the $B_s \rightarrow J/\psi \phi$ observables with the help of (71) and (72). Using in addition the observables involving the third linear polarization state f' that we have not employed so far, we can also fix its hadronic parameters $a_{f'}$ and $\theta_{f'}$, as well as the strong phase difference $\Delta_{f',f}$. Comparing $\Delta_{f',f}$ with its $B_s \rightarrow J/\psi \phi$ counterpart $\Delta'_{f',f}$, we may obtain valuable insights into possible corrections to (71).

As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the B_s^0 - \overline{B}_s^0 mixing phase ϕ_s from $B_s \rightarrow J/\psi \phi$. Although the penguin contributions are strongly suppressed in this mode because of the tiny parameter $\epsilon = 0.05$ (see (67)), they may well lead to uncertainties of the extracted value of ϕ_s at the level of 10%, since $\phi_s = \mathcal{O}(0.03)$ within the Standard Model. A measurement of $\phi_s = -2\lambda^2\eta$ would allow us to determine the Wolfenstein parameter η [17], thereby fixing the height of the unitarity triangle. Since the decay $B_s \rightarrow J/\psi \phi$ is very accessible at “second-generation” B -physics experiments performed at hadron machines, for instance at LHCb, it is an important issue to think about the hadronic uncertainties affecting the determination of ϕ_s from the corresponding angular distribution. The approach discussed above allows us to control these uncertainties with the help of $B_d \rightarrow J/\psi \rho^0$.

The experimental feasibility of the determination of γ from the $B_d \rightarrow J/\psi \rho^0$ angular distribution depends strongly on the “penguin parameters” a_f . It is very difficult to estimate these quantities theoretically. In contrast to the “usual” QCD penguin topologies, the QCD penguins contributing to $B_d \rightarrow J/\psi \rho^0$ require a colour-singlet exchange, i.e. are “Zweig-suppressed”. Such a comment does not apply to the electroweak penguins, which contribute in “colour-allowed” form. The current-current amplitude $A_{cc}^{(c)f}$ originates from “colour-suppressed” topologies, and the ratio $A_{\text{pen}}^{(ut)f} / [A_{cc}^{(c)f} + A_{\text{pen}}^{(ct)f}]$, which governs a_f , may be sizeable. It would be very important to have a better theoretical understanding of the quantities $a_f e^{i\theta_f}$. However, such analyses are far beyond the scope of this paper, and are left for further studies.

If the parameters a_f should all be very small, which would be indicated by $A_D^f = R_D = I_D^f = 0$, we could still determine the B_d^0 - \overline{B}_d^0 mixing phase from the observables $\hat{A}_M^f = \eta_f \sin \phi_d$. If we use, in addition, $\hat{I}_M^f = \cos \Delta_{f,\perp} \cos \phi_d$ and fix $\cos \Delta_{f,\perp}$ through the corresponding $B_s \rightarrow J/\psi \phi$ observable, $\cos \phi_d$ can be determined as well. Consequently, the B_d^0 - \overline{B}_d^0 mixing phase ϕ_d can be fixed *unambiguously* this way, thereby resolving a twofold ambiguity, which arises in the extraction of ϕ_d from $B_d \rightarrow J/\psi K_S$. This mode probes only $\sin \phi_d$. Since $\phi_d = 2\beta$, we are left with a twofold ambiguity for $\beta \in [0^\circ, 360^\circ]$. If we assume that $\beta \in [0^\circ, 180^\circ]$, as implied by the measured value of ε_K , we can fix β unambiguously. For alternative methods to deal with ambiguities of this kind, see [15].

Before we turn to $B_d \rightarrow \rho\rho$ and $B_s \rightarrow K^* \overline{K}^*$ decays, let us note that the approach presented in this section can also be applied to the angular distributions of the decay products of $B_{s(d)} \rightarrow J/\psi [\rightarrow l^+ l^-] K^* [\rightarrow \pi^0 K_S]$ and $B_{d(s)} \rightarrow D_{d(s)}^{*+} D_{d(s)}^{*-}$. For the $B_{s(d)} \rightarrow J/\psi K_S$ and $B_{d(s)} \rightarrow D_{d(s)}^+ D_{d(s)}^-$ variants of these strategies, see [16].

5 Further Applications

In this section, we discuss further applications of the general strategies presented in Section 3. All of the methods discussed below have counterparts using $B_{d,s}$ decays into two pseudoscalar mesons. If we replace the pseudoscalars by higher resonances, for example, by vector mesons, as in the following discussion, the angular distributions of their decay products provide interesting alternative ways to extract CKM phases and hadronic parameters, going beyond the $B_{d,s} \rightarrow PP$ strategies. Because of the many observables provided by the angular distributions, we can, moreover, perform many interesting cross checks, for example, of certain flavour-symmetry relations.

5.1 The Decays $B_d \rightarrow \rho^+ \rho^-$ and $B_s \rightarrow K^{*+} K^{*-}$

The decay $B_d^0 \rightarrow \rho^+ \rho^-$ originates from $\bar{b} \rightarrow \bar{u} u \bar{d}$ quark-level processes. Using the same notation as in (62), we have

$$A(B_d^0 \rightarrow [\rho^+ \rho^-]_f) = \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_f e^{i\gamma} \left[1 - d_f e^{i\Theta_f} e^{-i\gamma}\right], \quad (86)$$

where

$$\mathcal{C}_f \equiv \lambda^3 A R_b \left[\tilde{A}_{\text{cc}}^{(u)f} + \tilde{A}_{\text{pen}}^{(ut)f}\right] \quad (87)$$

and

$$d_f e^{i\Theta_f} \equiv \frac{1}{(1 - \lambda^2/2) R_b} \left[\frac{\tilde{A}_{\text{pen}}^{(ct)f}}{\tilde{A}_{\text{cc}}^{(u)f} + \tilde{A}_{\text{pen}}^{(ut)f}}\right]. \quad (88)$$

In order to distinguish the $B_d^0 \rightarrow \rho^+ \rho^-$ amplitudes from the $B_d^0 \rightarrow J/\psi \rho^0$ case discussed in the previous section, we have introduced the tildes. The phase structure of the $B_d^0 \rightarrow \rho^+ \rho^-$ decay amplitude given in (86), which is an exact parametrization within the Standard Model, is completely analogous to the one for the $B_d^0 \rightarrow \pi^+ \pi^-$ amplitude given in [22], where a more detailed discussion can be found.

The expressions for the observables describing the time evolution of the angular distribution of the decay products of $B_d^0 \rightarrow \rho^+ [\rightarrow \pi^+ \pi^0] \rho^- [\rightarrow \pi^- \pi^0]$ can be obtained straightforwardly from the formulae given in Section 2, by performing the following substitutions:

$$\mathcal{N}_f = \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_f, \quad b_f = d_f, \quad \rho_f = \Theta_f, \quad \omega = -\gamma. \quad (89)$$

Because of the factor of $e^{i\gamma}$ in front of the square brackets on the right-hand side of (86), we have to deal with a small complication. Since the observables are governed by

$$\xi_f = e^{-i\phi_d} \frac{\bar{A}_f}{A_f} = \eta_f e^{-i(\phi_d + 2\gamma)} \left[\frac{1 - d_f e^{i\Theta_f} e^{+i\gamma}}{1 - d_f e^{i\Theta_f} e^{-i\gamma}}\right], \quad (90)$$

we have to do the following replacement, in addition:

$$\phi_d \rightarrow \phi_d + 2\gamma. \quad (91)$$

Let us now turn to the decay $B_s^0 \rightarrow K^{*+}K^{*-}$, which is due to $\bar{b} \rightarrow \bar{u}u\bar{s}$ quark-level transitions. For a given final-state configuration f of the $K^{*+}K^{*-}$ pair, its decay amplitude can be parametrized as follows:

$$A(B_s^0 \rightarrow [K^{*+}K^{*-}]_f) = \lambda \mathcal{C}'_f e^{i\gamma} \left[1 + \left(\frac{1 - \lambda^2}{\lambda^2} \right) d'_f e^{i\Theta'_f} e^{-i\gamma} \right], \quad (92)$$

where \mathcal{C}'_f and $d'_f e^{i\Theta'_f}$ take the same form as (87) and (88), respectively, and the primes have been introduced to remind us that we are dealing with a $\bar{b} \rightarrow \bar{s}$ mode. The phase structure of (92) is completely analogous to the $B_s^0 \rightarrow K^+K^-$ decay amplitude [22]. The observables of the time-dependent $B_s \rightarrow K^{*+}[\rightarrow \pi K] K^{*-}[\rightarrow \bar{\pi} \bar{K}]$ angular distribution can be obtained straightforwardly from the formulae given in Section 2 by simply using the replacements:

$$\mathcal{N}_f = \lambda \mathcal{C}'_f, \quad b_f = \left(\frac{1 - \lambda^2}{\lambda^2} \right) d'_f, \quad \rho_f = \Theta'_f + 180^\circ, \quad \omega = -\gamma. \quad (93)$$

Moreover, we have to perform the substitution

$$\phi_s \rightarrow \phi_s + 2\gamma \quad (94)$$

because of the factor of $e^{i\gamma}$ in front of the square brackets in (92).

Explicit expressions for the $B_d \rightarrow \rho^+\rho^-$ and $B_s \rightarrow K^{*+}K^{*-}$ angular distributions in terms of helicity amplitudes can be found in [12]. Since $B_d^0 \rightarrow \rho^+\rho^-$ and $B_s^0 \rightarrow K^{*+}K^{*-}$ are related to each other by interchanging all down and strange quarks, the U -spin flavour symmetry of strong interactions implies

$$|\mathcal{C}'_f| = |\mathcal{C}_f|, \quad d'_f = d_f, \quad \Theta'_f = \Theta_f, \quad (95)$$

as well as

$$\Delta'_{\tilde{f},f} = \Delta_{\tilde{f},f}. \quad (96)$$

In contrast to (70)–(72), these relations do not rely on any dynamical assumption – just on the U -spin flavour symmetry. They can be used to combine the $B_d \rightarrow \rho^+\rho^-$ and $B_s \rightarrow K^{*+}K^{*-}$ observables with each other, thereby allowing the extraction of the CKM angle γ and of the $B_{d,s}^0 - \overline{B_{d,s}^0}$ mixing phases $\phi_d = 2\beta$ and $\phi_s = -2\delta\gamma$. In contrast to the $B_d \rightarrow \pi^+\pi^-$, $B_s \rightarrow K^+K^-$ variant of this approach proposed in [22], *both* mixing phases and the CKM angle γ can in principle be determined simultaneously. However, for the extraction of γ , it is more favourable to fix ϕ_d and ϕ_s separately. Then we are in a position to determine two contours in the $\gamma - \Delta_{\tilde{f},f}$ and $\gamma - \Delta'_{\tilde{f},f}$ planes in a *theoretically clean* way with the help of (53). Using now the U -spin relation (96), γ and all hadronic parameters describing the decays $B_d \rightarrow \rho^+\rho^-$ and $B_s \rightarrow K^{*+}K^{*-}$ can be determined. As we have already noted, the hadronic parameters provide a very fertile testing ground for model calculations of the decays $B_d \rightarrow \rho^+\rho^-$ and $B_s \rightarrow K^{*+}K^{*-}$. In particular, the penguin parameters $d_f e^{i\Theta_f}$ and $d'_f e^{i\Theta'_f}$ would be very interesting; comparing their

values with each other, we could obtain valuable insights into U -spin-breaking corrections. Moreover, there is one strong phase difference $\Delta'_{f',f}$ left, which can be compared with its U -spin counterpart $\Delta'_{f,f}$. If we should find a small difference between these phases, it would be quite convincing to assume that our U -spin input (96) is also not affected by large corrections.

Let us finally note that there is another interesting way to parametrize the $B_d \rightarrow \rho^+ \rho^-$ decay amplitudes (see also [23]). If we eliminate $\lambda_c^{(d)}$ through the unitarity of the CKM matrix – instead of $\lambda_t^{(d)}$, as done in (86) – we obtain

$$A(B_d^0 \rightarrow [\rho^+ \rho^-]_f) = \left(1 - \frac{\lambda^2}{2}\right) \lambda^3 A R_b e^{i\gamma} \left[\tilde{A}_{\text{cc}}^{(u)f} + \tilde{A}_{\text{pen}}^{(uc)f} \right] \left[1 + r_f e^{i\sigma_f} e^{-i(\beta+\gamma)} \right], \quad (97)$$

where

$$r_f e^{i\sigma_f} \equiv \frac{R_t}{(1 - \lambda^2/2) R_b} \left[\frac{\tilde{A}_{\text{pen}}^{(tc)f}}{\tilde{A}_{\text{cc}}^{(u)f} + \tilde{A}_{\text{pen}}^{(uc)f}} \right], \quad (98)$$

with

$$R_t \equiv \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| = \mathcal{O}(1). \quad (99)$$

Taking into account that we have $\phi_d = 2\beta$ and $\beta + \gamma = 180^\circ - \alpha$ within the Standard Model, we arrive at

$$b_f = r_f, \quad \rho_f = \sigma_f, \quad \omega = \alpha, \quad (100)$$

and at the “effective” mixing phase $\phi = \phi_d + 2\gamma = -2\alpha$. Consequently, using the strategy presented in Section 3, the $B_d \rightarrow \rho^+ \rho^-$ angular distribution allows us to probe also the combination $\alpha = 180^\circ - \beta - \gamma$ directly, i.e. to determine α as a function of a CP-conserving strong phase difference $\Delta_{\tilde{f},f}$ (see also (58) and (59)). Needless to note that the decay $B_d \rightarrow \rho^0 \rho^0$ may also be interesting in this respect. Since the normalization factors \mathcal{N}_f of the parametrization (97) are proportional to

$$\tilde{A}_{\text{cc}}^{(u)f} + \tilde{A}_{\text{pen}}^{(uc)f}, \quad (101)$$

which is governed by “colour-allowed tree-diagram-like” topologies, it may well be that $\Delta_{\tilde{f},f} \approx 0$. This relation would allow us to extract α , as well as the $B_d \rightarrow \rho^+ \rho^-$ hadronic parameters, which include also another strong phase difference $\Delta'_{f',f}$, providing an important cross check.

5.2 The Decays $B_d \rightarrow K^{*0} \overline{K^{*0}}$ and $B_s \rightarrow K^{*0} \overline{K^{*0}}$

The decays $B_d^0 \rightarrow K^{*0} \overline{K^{*0}}$ and $B_s^0 \rightarrow K^{*0} \overline{K^{*0}}$ are pure “penguin” modes, originating from $\bar{b} \rightarrow \bar{d} s \bar{s}$ and $\bar{b} \rightarrow \bar{s} d \bar{d}$ quark-level transitions, respectively. They do not receive contributions from current–current operators at the “tree” level, and can be parametrized within the Standard Model in complete analogy to (62) and (67). We have just to set the current–current amplitudes equal to zero in these expressions. The decays $B_d^0 \rightarrow K^{*0} \overline{K^{*0}}$ and $B_s^0 \rightarrow K^{*0} \overline{K^{*0}}$ are related to each other by interchanging all down and strange

quarks, i.e. through the U -spin flavour symmetry of strong interactions, and the strategies to probe γ and the $B_{d,s}^0 - \overline{B}_{d,s}^0$ mixing phases are analogous to those discussed in Section 4. Since the $B_{d,s} \rightarrow K^{*0} \overline{K}^{*0}$ decays are pure “penguin” modes, they represent a particularly sensitive probe for new physics.

An interesting alternative to parametrize the $B_d^0 \rightarrow [K^{*0} \overline{K}^{*0}]_f$ decay amplitudes within the Standard Model is given as follows:

$$A(B_d^0 \rightarrow [K^{*0} \overline{K}^{*0}]_f) = \lambda^3 A R_t \mathcal{A}_{\text{pen}}^{(tu)f} e^{-i\beta} [1 - g_f e^{i\varphi_f} e^{i\beta}], \quad (102)$$

where

$$g_f e^{i\varphi_f} \equiv \frac{1}{R_t} \frac{\mathcal{A}_{\text{pen}}^{(cu)f}}{\mathcal{A}_{\text{pen}}^{(tu)f}} \quad (103)$$

may well be sizeable due to the presence of final-state-interaction effects [24]. Consequently, we have

$$b_f = g_f, \quad \rho_f = \varphi_f, \quad \omega = \beta. \quad (104)$$

Because of the factor of $e^{-i\beta}$ in front of the square brackets on the right-hand side of (102), the “effective” mixing phase is given by $\phi = \phi_d - 2\beta$. Consequently, the strategy presented in Section 3 allows us to probe the CP-violating weak phase β of the CKM element $V_{td} = |V_{td}|e^{-i\beta}$. Within the Standard Model, we have $\phi = \phi_d - 2\beta = 0$. However, this relation may well be affected by new physics, and represents a powerful test of the Standard-Model description of CP violation (for a recent discussion, see [25]). Therefore it would be very important to determine this combination of CKM phases experimentally. The observables of the $B_d \rightarrow K^{*0}[\rightarrow \pi^- K^+] \overline{K}^{*0}[\rightarrow \pi^+ K^-]$ angular distribution may provide an important step towards this goal.

6 Conclusions

The angular distributions of certain quasi-two-body modes $B_{d,s} \rightarrow X_1 X_2$, where both X_1 and X_2 carry spin and continue to decay through CP-conserving interactions, provide valuable information about CKM phases and hadronic parameters. We have presented the general formalism to accomplish this task, taking into account also penguin contributions, and have illustrated it by having a closer look at a few specific decay modes. In comparison with strategies using non-leptonic $B_{d,s}$ decays into two pseudoscalar mesons, an important advantage of the angular distributions is that they provide much more information, thereby allowing various interesting cross checks, for instance, of certain flavour-symmetry relations. Moreover, they provide a very fertile testing ground for model calculations of the $B_{d,s} \rightarrow X_1 X_2$ modes.

We have pointed out that the decay $B_d \rightarrow J/\psi \rho^0$ can be combined with $B_s \rightarrow J/\psi \phi$ to extract the $B_d^0 - \overline{B}_d^0$ mixing phase $\phi_d = 2\beta$ and – if penguin effects in the former mode should be sizeable – also the angle γ of the unitarity triangle. As an interesting by-product, this strategy allows us to take into account also the penguin effects in the extraction of the $B_s^0 - \overline{B}_s^0$ mixing phase from $B_s \rightarrow J/\psi \phi$. If penguin effects should be very

small in $B_d \rightarrow J/\psi \rho^0$, ϕ_d could still be determined and it would even be possible to resolve a twofold ambiguity, arising in the extraction of this CKM phase from $B_d \rightarrow J/\psi K_S$. Other interesting applications, involving $B_d \rightarrow \rho\rho$ and $B_{s,d} \rightarrow K^* \overline{K}^*$ decays, were also noted. Within the Standard Model, these modes are expected to exhibit branching ratios at the 10^{-5} level; also the one for $B_d \rightarrow K^{*0} \overline{K}^{*0}$ may well be enhanced, from its “short-distance” expectation of $\mathcal{O}(10^{-6})$ to this level, by final-state-interaction effects.

Since the formalism presented in this paper is very general, it can of course be applied to many other decays. Detailed studies are required to explore which channels are most promising from an experimental point of view. Although the B_d modes listed above may already be accessible at the asymmetric e^+e^- B -factories operating at the $\Upsilon(4S)$ resonance, which will start taking data very soon, the strategies presented in this paper appear to be particularly interesting for “second-generation” experiments at hadron machines, such as LHCb or BTeV, where also the very powerful physics potential of the B_s system can be exploited.

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